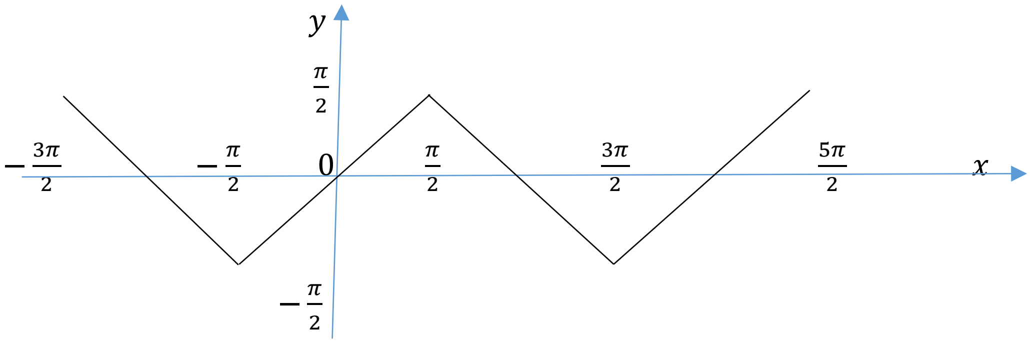
Profesor Blaga Mirela-Gabriela

Compunerea funcţiilor trigonometrice - 2

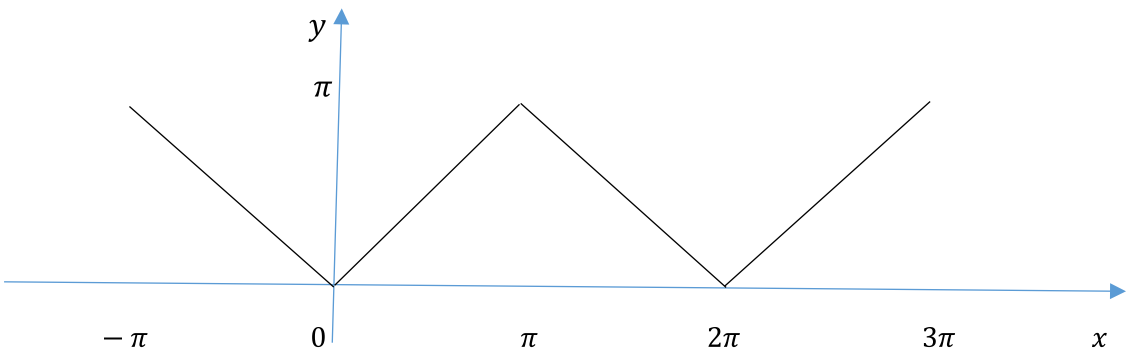
1. $f:\mathbb{R}\to\left[-\frac{\pi}{2},\frac{\pi}{2}\right], f(x)=arcsin(sinx)$

$$f(x)=arcsin(sinx)=\begin{cases} \dots \\ x & ,x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right) \\ \pi-x & ,x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right) \\ x-2\pi & ,x\in\left[\frac{3\pi}{2},\frac{5\pi}{2}\right) \\ \dots \end{cases}$$



2. $f:\mathbb{R}\to[0,\pi], f(x)=arccos(cosx)$

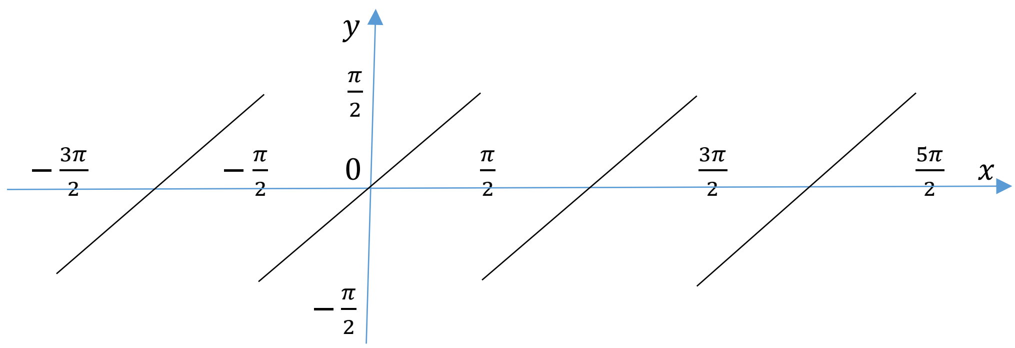
$$f(x)=arccos(cosx)=\begin{cases} \dots \\ 2\pi+x & ,x\in[-2\pi,-\pi) \\ -x & ,x\in[-\pi,0) \\ x & ,x\in[0,\pi) \\ 2\pi-x & ,x\in[\pi,2\pi) \\ \dots \end{cases}$$



$3. f:\mathbb{R}\backslash\left\{\frac{(2k+1)\pi}{2}\middle|k\in\mathbb{Z}\right\}\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right), f(x)=arctg(tgx)$

$f(x)=x-k\pi, x\in\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi\right), k\in\mathbb{Z}$

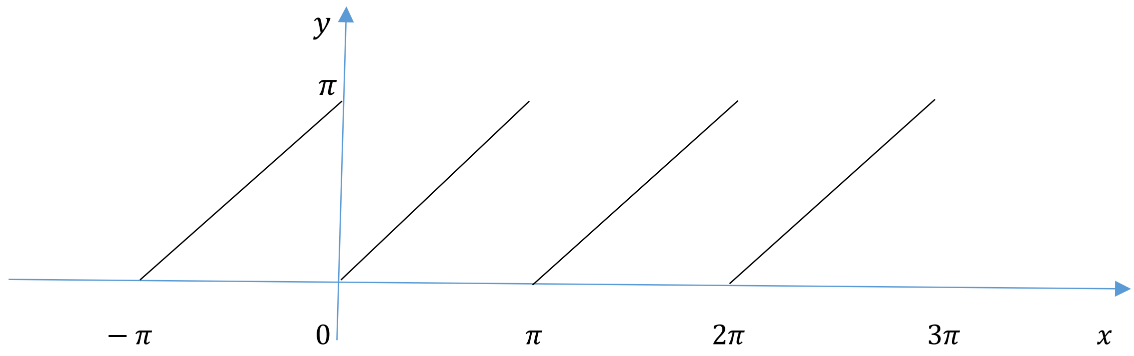
$$f(x)=arctg(tgx)=\begin{cases} \dots \\ x+\pi & ,x\in\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right) \\ x & ,x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \\ x-\pi & ,x\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right) \\ x-2\pi & ,x\in\left(\frac{3\pi}{2},\frac{5\pi}{2}\right) \\ \dots \end{cases}$$



$4. f:\mathbb{R}\backslash\{k\pi|k\in\mathbb{Z}\}\to(0,\pi), f(x)=arcctg(ctgx)$

$f(x)=x-k\pi, x\in(k\pi,\pi+k\pi), k\in\mathbb{Z}$

$$f(x)=arcctg(ctgx)=\begin{cases} \dots \\ x+\pi & ,x\in(-\pi,0) \\ x & ,x\in(0,\pi) \\ x-\pi & ,x\in(\pi,2\pi) \\ x-2\pi & ,x\in(2\pi,3\pi) \\ \dots \end{cases}$$



Aplicații

1)Calculați $arcsin\left(sin\frac{\pi}{4}\right)$ și $arcsin\left(sin\frac{3\pi}{4}\right)$.

$\frac{\pi}{4}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right) \ \rightarrow \ arcsin\left(sin\frac{\pi}{4}\right)=\frac{\pi}{4}$

$\frac{3\pi}{4}\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right) \ \rightarrow \ arcsin\left(sin\frac{3\pi}{4}\right)=\pi-\frac{3\pi}{4}=\frac{\pi}{4}$

2)Fie funcția $f:\mathbb{R}\rightarrow\mathbb{R}, f(x)=arcsin(sinx)$.

a)Aflați f'.

b)Calculați $\int_0^{\frac{5\pi}{2}} f(x)dx$.

a) $f':\mathbb{R}\backslash\left\{\frac{(2k+1)\pi}{2}\middle|k\in\mathbb{Z}\right\}\rightarrow\mathbb{R}, f'(x)=\begin{cases}\dots\\ 1 & ,x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\\ -1 & ,x\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\\ 1 & ,x\in\left(\frac{3\pi}{2},\frac{5\pi}{2}\right)\\ \dots\end{cases}$

b) $\int_0^{\frac{5\pi}{2}} f(x)dx=\int_0^{\frac{\pi}{2}} f(x)dx+\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(x)dx+\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} f(x)dx=$

$=\int_0^{\frac{\pi}{2}} xdx+\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(\pi-x)dx+\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}}(x-2\pi)dx=$

$=\frac{x^2}{2}\Big|_0^{\frac{\pi}{2}}+\left(\pi x-\frac{x^2}{2}\right)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}+\left(\frac{x^2}{2}-2\pi x\right)\Big|_{\frac{3\pi}{2}}^{\frac{5\pi}{2}}=\frac{\pi^2}{8}$

3)Calculați $arctg\frac{1}{3}+arctg\frac{1}{7}+arctg\frac{1}{13}+\ \dots$.

Fie $a_1=3, a_2=7, a_3=13$ primii trei termeni ai unui șir.

Scriem suma S a primilor n termeni și efectuăm diferența pentru a obține a_n.

$$
\begin{array}{l}
S = 3 + 7 + 13 + 21 + \ \ldots + a_n \\
S = \qquad 3 + \ 7 + 13 + \ \ldots + a_{n-1} + a_n \\
\hline
0 = 3 + (7-3) + (13-7) + \ \ldots + (a_n - a_{n-1}) - a_n
\end{array}
$$

$$a_n = 3 + \underbrace{4 + 6 + 8 + \ \ldots + (a_n - a_{n-1})}_{n-1 \text{ termeni în progresie aritmetică}}$$

$$a_n = 3 + \frac{(n-1)[2 \cdot 4 + (n-1-1) \cdot 2]}{2}$$

$$a_n = n^2 + n + 1$$

Termenul general este

$$arctg \frac{1}{n^2+n+1} = arctg \frac{n+1-n}{1+n(n+1)} = arctg(n+1) - arctg\, n.$$

$$arctg \frac{1}{3} + arctg \frac{1}{7} + arctg \frac{1}{13} + \ \ldots = \lim_{n \to \infty} \sum_{k=1}^{n} (arctg(k+1) - arctg\, k) =$$

$$= \lim_{n \to \infty} (arctg(n+1) - arctg\ 1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$