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Limite remarcabile de tipul $\frac{0}{0}$. Aplicații

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

Din limita remarcabilă precedentă deducem:

1. $\lim_{x\to 0}\frac{\sin ax}{x}=a \qquad ,a\in\mathbb{R}$

2. $\lim_{x\to 0}\frac{\sin ax}{bx}=\frac{a}{b} \qquad ,a\in\mathbb{R}, b\in\mathbb{R}^*$

3. $\lim_{x\to 0}\frac{\sin^2 x}{x^2}=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2=1$

4. $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

Prima variantă de rezolvare a limitei cu numărul 4:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{0}{0}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}=\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}=\frac{1}{2}$$

A doua variantă de rezolvare a limitei cu numărul 4:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{0}{0}=\lim_{x\to 0}\frac{2\cdot\sin^2\frac{x}{2}}{x^2}=\lim_{x\to 0}\frac{2\cdot\sin^2\frac{x}{2}}{\frac{x^2}{4}\cdot 4}=\frac{2}{4}=\frac{1}{2}$$

A treia variantă de rezolvare a limitei cu numărul 4:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{0}{0}\overset{l'Hospital}{=}\lim_{x\to 0}\frac{\sin x}{2x}=\frac{1}{2}$$

A patra variantă de rezolvare a limitei cu numărul 4:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{0}{0}\overset{Polinom\ Taylor}{=}\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ \dots\right)}{x^2}=$$

$$=\lim_{x\to 0}\frac{\frac{x^2}{2!}-\frac{x^4}{4!}+\frac{x^6}{6!}-\ \dots}{x^2}=\lim_{x\to 0}\frac{x^2\cdot\left(\frac{1}{2!}-\frac{x^2}{4!}+\frac{x^4}{6!}-\ \dots\right)}{x^2}=\frac{1}{2!}=\frac{1}{2}$$

5. $\lim_{x\to 0}\frac{x-\sin x}{x^3}=\frac{1}{6}$

6. $\lim_{x\to 0}\frac{x^n-\sin x^n}{x^{3n}}=\frac{1}{6} \qquad ,n\in\mathbb{N}^*$

7. $\lim\limits_{x \to 0} \frac{x^n - sin^n x}{x^{n+2}} = \frac{n}{6}$ $\quad , n \in \mathbb{N}$

O metodă de rezolvare a limitei cu numărul 7 este:

$$\lim\limits_{x \to 0} \frac{x^n - sin^n x}{x^{n+2}} = \lim\limits_{x \to 0} \frac{(x - sinx)(x^{n-1} + x^{n-2} \cdot sinx + \ \dots + sin^{n-1}x)}{x^{n+2}} =$$

$$= \lim\limits_{x \to 0} \frac{(x - sinx)(x^{n-1} + x^{n-2} \cdot sinx + \ \dots + sin^{n-1}x)}{x^3 \cdot x^{n-1}} =$$

$$= \lim\limits_{x \to 0} \frac{x - sinx}{x^3} \cdot \lim\limits_{x \to 0} \frac{\overbrace{x^{n-1} + x^{n-2} \cdot sinx + \ \dots + sin^{n-1}x}^{n \ termeni}}{x^{n-1}} =$$

$$= \frac{1}{6} \cdot n = \frac{n}{6}$$

8. $\lim\limits_{x \to 0} \frac{\sqrt[3]{x - sinx}}{x} = \lim\limits_{x \to 0} \sqrt[3]{\frac{x - sinx}{x^3}} = \frac{1}{\sqrt[3]{6}}$

9. $\lim\limits_{x \to 0} \frac{sinx + sin2x + sin3x + \ \dots \ + sinnx}{x} = 1 + 2 + 3 + \ \dots \ + n = \frac{n(n+1)}{2}$ $\quad , n \in \mathbb{N}$

10. $\lim\limits_{x \to 0} \frac{1 - cosx \cdot cos2x \cdot cos3x \cdot \dots \cdot cosnx}{x^2} = \frac{n(n+1)(2n+1)}{12}$ $\quad , n \in \mathbb{N}$

11. $\lim\limits_{x \to x_0} \frac{sin(x - x_0)}{x - x_0} \overset{\text{notăm } x - x_0 = y}{=} \lim\limits_{y \to 0} \frac{siny}{y} = 1$

12. $\lim\limits_{x \to \infty} x \cdot sin\frac{1}{x} \overset{\text{notăm } \frac{1}{x} = y}{=} \lim\limits_{y \to 0} \frac{siny}{y} = 1$

Sursa https://it.wikipedia.org/wiki/Limite_notevole